Referee report on the thesis
"The Total Least Squares Problem and Reduction of Data in $AX \approx B$"
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This thesis considers an orthogonally invariant linear approximation problem with multiple right-hand sides $AX \approx B$ in the context of the Total Least Squares (TLS) problem, which aims at minimizing corrections on A and B that make the system consistent. This fundamental matrix problem has been studied for many decades and applications are known in many fields. However, a discrepancy in the study of the TLS problem has been the fact that a complete classification of the TLS problems was only available for the *single right-hand side* case; moreover, only for this case the *core problem theory*, which studies the reduction of data to the minimal necessary and sufficient information needed to solve the original TLS problem, has been recently developed. This thesis sets important first steps into the study of the *multiple right-hand side* TLS problem and the associated core problem theory.

The thesis is split into four parts. The first part, the introduction, reviews the single right-hand side TLS problem and the core problem theory in this particular case. This up-to-date review is completed, interestingly enough, with a section devoted to an "alternative core problem definition", which is motivated by the work on the multiple right-hand side case presented further in the thesis. The introduction opens the gates towards the main topics of the thesis, developed in parts II and III: the *analysis of the TLS problem with multiple right-hand side* and the *data reduction in* $AX \approx B$, respectively.

The main contribution of Part II is a complete characterization of all cases that can arise in relation to the TLS problem with multiple right-hand side. Existence and uniqueness conditions for the TLS solution, as well as a thorough analysis of the cases when the TLS solution cannot exist are presented. The author brings to light a much-needed classification of the multiple right-hand side approximation problems, refining thus the splitting proposed by Van Huffel and Vandewalle into *generic* and *nongeneric* TLS problems. The new classification is based on intricate conditions on the distribution of singular values of $[B|A]$, as well as rank conditions on subblocks of the right singular matrix. It is important to notice that the most general distribution of singular values was excluded, for convenience, from the initial analysis of Van Huffel and Vandewalle, but it is studied in depth in the thesis. This study leads to interesting ideas, most notably the (counter-intuitive) conclusion that the *TLS algorithm* could compute a non-optimal solution even in certain cases when a TLS solution actually exists.

Part III extend the concept of core problem within a linear approximation problem from the single right-hand side case to the multiple right-hand side case. Although defining the reduction problem is a straightforward generalization of the definition used in the single right-hand side case, the computational methods performing this data reduction are essentially different from the ones used in the single right-hand side case. Two algorithms are proposed, the first being based on singular value decompositions, the second on reduction to band structured matrices. The reduced problems obtained by both algorithms satisfy the same general properties; these properties become an alternative definition for the core problem.

In the process of developing the second algorithm (a band generalization of the Golub-Kahan bidiagonalization) the author introduces a novel matrix concept: the *wedge-shaped matrix*; he proves various properties of this type of matrices, which allow them to become useful tools in the analysis of the banded form of the core problem.

Finally, the core problem is studied in relation to the TLS problem classification. As opposed to the single right-hand side case, the core problem in the general case is not guaranteed to have optimal and unique TLS solution. This rather unsettling observation gives rise to a deeper analysis of the core problem, and yields a classification of core problems into decomposable or non-decomposable core problems. The issue of revealing independent subproblems (revealing what the author calls "the chessboard structure") within the core problem is however a difficult topic. The author makes a fundamental observation that applying the TLS algorithm on the independent subproblems of a decomposable problem, and combining the solutions, can yield different results than applying the TLS algorithm to the problem as a whole.

Part IV presents numerical examples and practical issues regarding the single right hand side core problem. What is missing from this dissertation is an analysis of the data reduction methods for multiple right-hand side problems in finite precision arithmetic, as well as details on numerical implementation, that could parallel the single right-hand side computational part. However, such an analysis and corresponding numerical experiments constitute a very broad topic in itself, therefore it is normal that they were not treated in this—mostly theoretical—work.

The analysis of a core reduction for $AX \approx B$ is a topic in its infancy; for this reason there are still many interesting open questions, especially in what concerns the TLS solution of the core problem. This thesis is a door-opener, which has the potential of attracting much interest in the matrix computations community, and it will probably give rise to further research in this area.

Conclusion

This thesis contains high quality original results presented with rigour and skill. The research abilities of the candidate are obvious. I highly recommend Ing. Martin Plešinger for the title of Doctor.

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