$$\left(\sum_{k=1}^{\infty} a_k(x)\right)' \overset{?}{=} \sum_{k=1}^{\infty} a_k'(x) = \lim_{h\to 0} \frac{a_k(x+h)-a_k(x)}{h}$$

$$\left(\sum_{k=1}^{n} a_k(x)\right)' = \sum_{k=1}^{n} a_k'(x)$$

$$\left(\lim_{n\to\infty}\sum_{k=1}^{n} a_k(x)\right)' = \lim_{h\to 0} \frac{\lim_{n\to\infty}\left(\sum_{k=1}^{n} a_k(x+h) - \cancel{\lim_{n\to\infty}}\sum_{k=1}^{n} a_k(x)\right)}{h} =$$

$$= \lim_{h\to 0}\lim_{n\to\infty}\sum_{k=1}^{n} \frac{a_k(x+h)-a_k(x)}{h}$$

$$\lim_{n\to\infty}\sum_{k=1}^{n} \lim_{h\to 0}\frac{a_k(x+h)-a_k(x)}{h} = \lim_{n\to\infty}\lim_{h\to 0}\frac{a_k(x+h)-a_k(x)}{h}$$