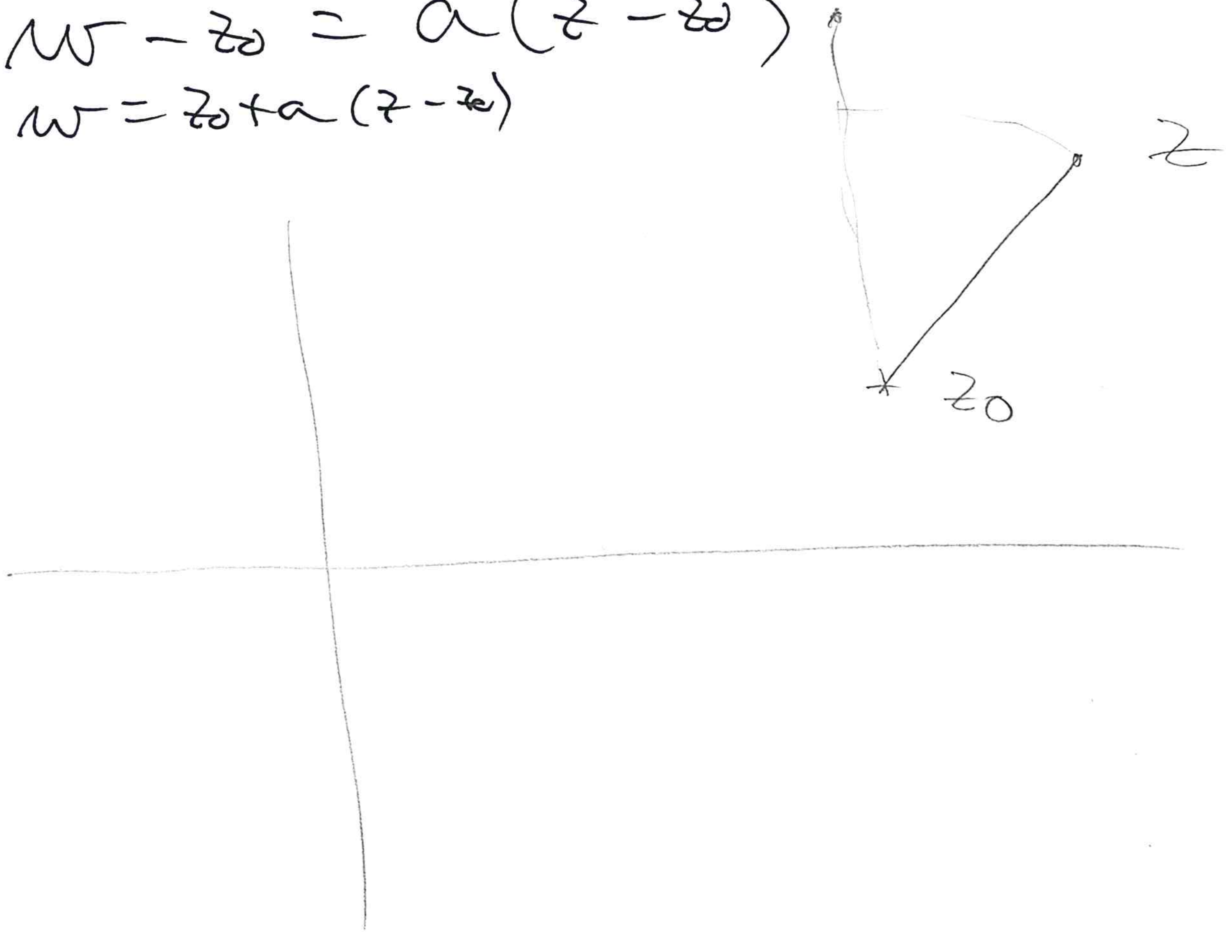
$w = az + b$ $\qquad a \neq 1$ $\qquad z_0 = az_0 + b$

$z_0 = az_0 + b$ $\qquad z_0 = \dfrac{b}{1-a}$

$w - z_0 = a(z - z_0)$

$w = z_0 + a(z - z_0)$



13 $f(z) = \frac{2}{z+2}$

$z_0 = 1$

$$\sum_{k=0}^{\infty} a_k (z-1)^k$$

$$\sum_{k=0}^{\infty} a_0 q^k = \frac{a_0}{1-q}$$

$$\frac{2 = a_0}{1 - \underbrace{2 \cdot (z-1)}_{=q}}$$

13a $f(z) = \frac{2}{z^2 - 3z} =$

$$= \frac{A}{z} + \frac{B}{z-3}$$

$$\sum a_k (z-1)^k \quad \sum b_k (z-1)^k$$

$$\sum (a_k + b_k)(z-1)^k$$

$$\frac{2}{z+2} = \frac{2 \cdot \frac{1}{3}}{\left((z-1)+3\right)\cdot\frac{1}{3}} = \frac{\frac{2}{3}}{1+\frac{1}{3}(z-1)}$$

$$a_0 = \frac{2}{3}$$

$$q = -\frac{1}{3}(z-1)$$

$$\sum_{k=0}^{\infty} \frac{2}{3}\left(-\frac{1}{3}\right)^k (z-1)^k$$

$$|q| < 1 \; : \quad \frac{1}{3}|z-1| < 1$$

$$\boxed{|z-1| < 3} = R$$

Co znamená absolutní konvergence řady

$$\sum_{k=0}^{\infty} a_k (z-z_0)^k \quad ?$$

$$\sum_{k=0}^{\infty} \left| a_k (z-z_0)^k \right| < +\infty$$

z_1

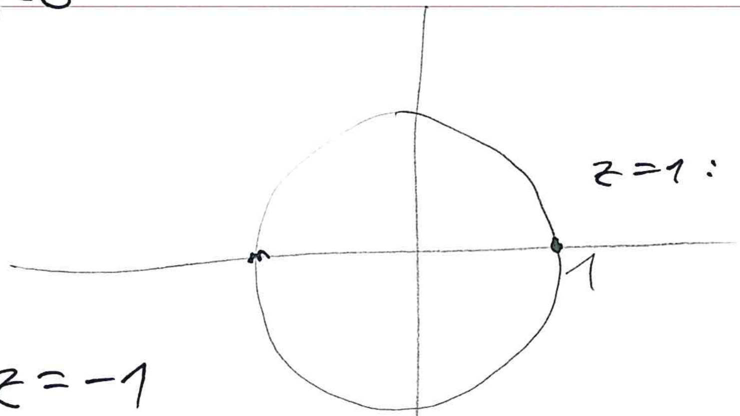
$\cdot\, z_1$ $\quad$ $\cdot\, z_2$

$\cdot\, z_0$

$$|z_2 - z_0| > |z_1 - z_0|$$

$$\sum_{k=1}^{\infty} \frac{1}{k} z^k$$

$z_0 = 0$



$z=1$: $\sum \frac{1}{k} = +\infty$

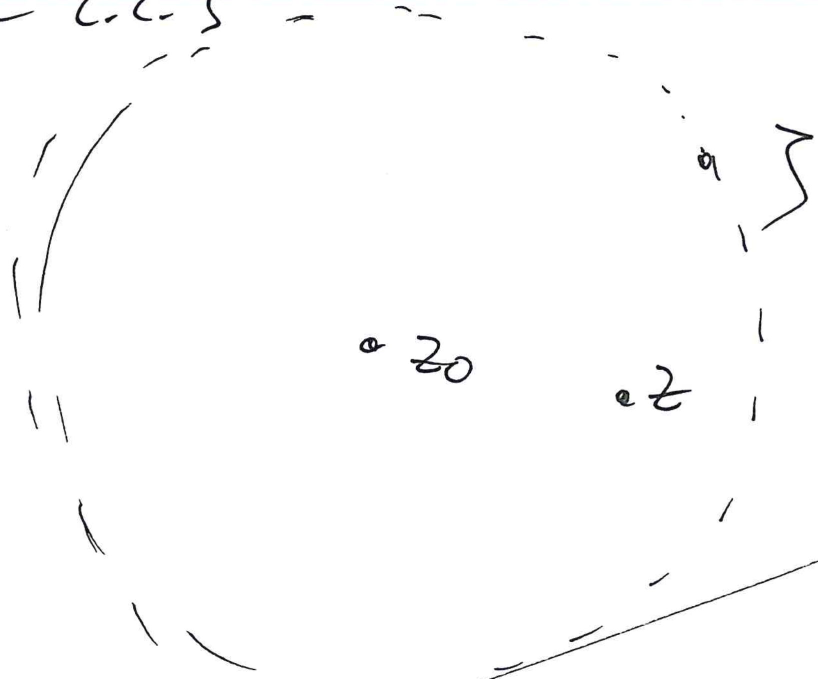
$z=-1$

$$\sum \frac{(-1)^k}{k} = -1 + \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \dots$$

~~neabs~~

konverguje, ale ~~ne~~ absolutně

Lema 2.2.3



$|z - z_0| < |\zeta - z_0|$

2.2 je rovnice kruhu

$$\sum_{k=1}^{\infty} \frac{1}{k} = +\infty$$

$$\sum_{k=1}^{\infty} \frac{1}{k^2} < +\infty \quad \left(\frac{\pi^2}{6}\right)$$

$$\left.\begin{array}{l} \displaystyle\sum_{k=1}^{\infty} \frac{1}{k} z^k \\ \\ \displaystyle\sum_{k=1}^{\infty} \frac{1}{k^2} z^k \end{array}\right\} R = 1$$

absolutní konvergence

pro $|z| = 1$

$$\sum \underbrace{\left|\frac{1}{k^2} z^k\right|}_{\frac{1}{k^2}} < +\infty$$

$$R = \lim_{k\to\infty}\left|\frac{a_k}{a_{k+1}}\right|$$ pokud limita existuje

$$\sum_{k=0}^{\infty} a_k\, k\,(z-z_0)^{k-1}$$

$$\left|\frac{a_k\cdot k}{a_{k+1}\cdot(k+1)}\right| = \underbrace{\left|\frac{a_k}{a_{k+1}}\right|}_{\to R}\cdot\underbrace{\frac{k}{k+1}}_{\to 1} \to R\cdot 1 = R$$

věta o limitě součinu

Věta 2.3.5

$$f(z) = \sum a_k (z-z_0)^k$$

$$g(z) = \sum a_k k (z-z_0)^{k-1}$$

$$f'(z) = g(z)$$

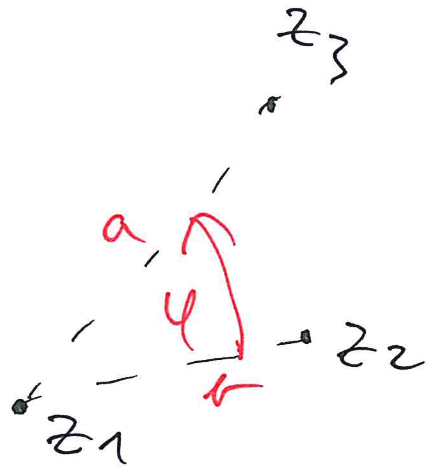
$$\left(\sum a_k (z-z_0)^k\right)' = \sum \left(a_k (z-z_0)^k\right)'$$

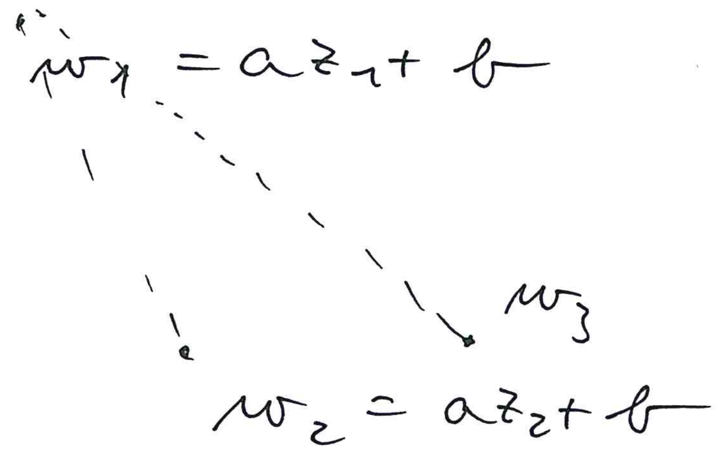
tohle není věta o derivaci součtu

$$\underbrace{\lim_{k\to\infty} \lim_{x\to 1^-} x^k}_{=1} \neq \underbrace{\lim_{x\to 1} \lim_{k\to\infty} x^k}_{=0}$$

Lineární funkce a podobné zobrazení

$$w = az + b$$





poměr bodů:
(vektorů)

$$\frac{z_3 - z_1}{z_2 - z_1} = \frac{a}{b}(\cos\varphi + i\sin\varphi)$$

potom obrazů:

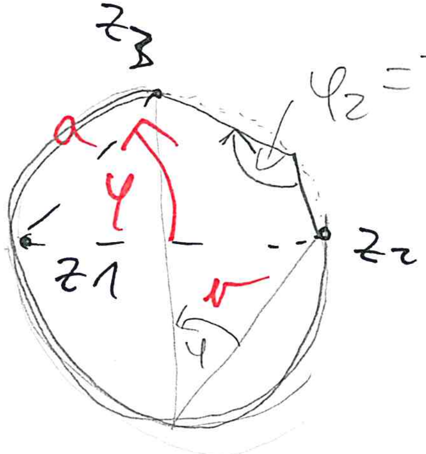
$$\frac{w_3 - w_1}{w_2 - w_1} = \frac{az_3 + b - (az_1 + b)}{az_2 + b - (az_1 + b)} = \frac{z_3 - z_1}{z_2 - z_1}$$

Dvojpoměr bodů

$$\frac{\dfrac{z_3 - z_1}{z_2 - z_1}}{\dfrac{z_3 - z_4}{z_2 - z_4}} \qquad \frac{z_3 - z_1}{z_2 - z_1} = \frac{a}{r}(\cos\varphi + i\sin\varphi)$$

Proč

Jak geometricky charakterizovat čtveřici bodů s reálnou hodnotou dvojpoměru:



opačně orientovaný

$\varphi_1 - \pi$

$$\cancel{\frac{\cos\varphi + i\sin\varphi}{\cos\varphi - i\sin\varphi}}$$

$$\cancel{\cos(-\varphi) + i\sin(-\varphi)}$$

$$\frac{\cos\varphi + i\sin\varphi}{\cos(\varphi - \pi) + i\sin(\varphi - \pi)} = -1$$

$\cdot z_1 \qquad \cdot z_2 \qquad \cdot z_3$

$$\frac{z_3 - z_1}{z_2 - z_1} \in \mathbb{R}$$

poměr bodů $z_1, z_2, z_3 \in \mathbb{R}$ $\Longleftrightarrow$ body leží na přímce

dvojpoměr $z_1, \ldots, z_4 \in \mathbb{R}$ $\Longleftrightarrow$ body leží na přímce nebo kružnici

$w = \frac{1}{z}$ zachovávání dvojpoměru

$w = \frac{az+b}{cz+d}$ — " —

$w = \frac{1}{\bar{z}}$